CREATION OF UNIVERSES FROM NOTHING

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A cosmological model is proposed in which the universe is created by quantum tunneling from literally nothing into a de Sitter space. After the tunneling, the model evolves along the lines of the inflationary scenario. This model does not have a big-bang singularity and does not require any initial or boundary conditions.

The standard hot cosmological model gives a successful description of many features of the evolution of the universe. However, it is not totally satisfactory, since it requires rather unnatural initial conditions at the big bang. One has to postulate that the universe has started in a homogeneous and isotropic state with tiny density fluctuations which are to evolve into galaxies. Homogeneity and isotropy must extend to scales far exceeding the causal horizon at the Planck time. In addition, the energy density of the universe must be tuned to be near the critical density with an incredible accuracy of $\sim 10^{-55}$.

In the last few years there is a growing hope of explaining these initial conditions as resulting from physical processes in the very early universe. Guth [1] has suggested that the homogeneity, isotropy and flatness puzzles can be solved if the universe passed through a de Sitter phase of exponential expansion (inflation) in its early history. [$a(t) \propto \exp(Ht)$, where $a(t)$ is the scale factor.] Such a phase can arise in a first order phase transition with strong supercooling. It has been suggested [2–4] that extreme supercooling can occur in grand unified models with Coleman–Weinberg type of symmetry breaking. Initially it was not clear how to end the exponential expansion and get back to a radiation-dominated universe [1–5].

A plausible answer has emerged quite recently [3,4,6]. At some temperature T_0 the false vacuum becomes unstable due to thermal [2–4] or gravitational [6,7] effects. The Higgs field ϕ starts rolling down the effective potential towards the absolute minimum, $\phi = \sigma$. The Coleman–Weinberg potential is very flat for small values of $\phi (\phi \ll \sigma)$, and the typical rollover time, τ, can be much greater than the expansion time, H^{-1}. Until ϕ becomes of the order σ, exponential expansion continues, and the scale of the universe grows by a factor $\sim \exp(H\tau) \ggg 1$. To solve the homogeneity and flatness problems we need $\exp(H\tau) \gtrsim 10^{28}$ [1]. Most of this growth takes place after the destabilization of the false vacuum. When ϕ becomes $\sim \sigma$, the vacuum energy thermalizes, and the universe enters a radiation-dominated period. The baryon number can be generated during the thermalization or shortly afterwards. Density fluctuations can be generated by vacuum strings produced at a later phase transition [8]. Another attractive feature of this scenario is that the problem of superabundance of heavy magnetic monopoles does not arise: the Higgs expectation value is uniform over the whole visible universe.

Now that we have a plausible ending to the inflationary scenario, we can start wondering about its beginning, where the situation is still rather depressing. There is a cosmological singularity at $t = 0$ and the origin of the initial thermal state is mysterious. Besides, there is another problem if we assume that the universe is closed (which seems to be a more aesthetically appealing choice). It is natural to assume that at about Planck time ($t \sim t_p$) the size and the energy density of the universe are $O(1)$ in Planck units. But then the universe will expand and recollapse in about one Planck time, its size will never much exceed the Planck length, and the phase of exponential expansion will never be reached (assuming that the grand unification mass scale is much smaller than the Planck mass, $\sigma \ll m_P$). In order to cool down to temperatures $\sim 10^{14}$ GeV, the energy density at $t \sim t_P$ must be tuned to be near the critical density with an accuracy of $\sim 10^{-10}$. This is just a milder version of the same flatness problem that we faced before.

In this paper I would like to suggest a new cosmological scenario in which the universe is spontaneously created from literally *nothing*, and which is free from the difficulties I mentioned in the preceding paragraph. This scenario does not require any changes in the fundamental equations of physics; it only gives a new interpretation to a well-known cosmological solution.

We shall consider a model of interacting gravitational and matter fields. The matter content of the model can be taken to be that of some grand unified theory (GUT). The absolute minimum of the effective potential is reached when the Higgs field ϕ responsible for the GUT symmetry breaking acquires a vacuum expectation value, $\langle\phi\rangle = \sigma \ll m_P$. The symmetric vacuum state, $\langle\phi\rangle = 0$, has a nonzero energy density, ρ_V. For a Coleman–Weinberg potential,

$$\rho_V \sim g^4\sigma^4 , \tag{1}$$

where g is the gauge coupling.

Suppose that the universe starts in the symmetric vacuum state and is described by a closed Robertson–Walker metric.

$$ds^2 = dt^2 - a^2(t)[dr^2/(1-r^2) + r^2 d\Omega^2] . \tag{2}$$

The scale factor $a(t)$ can be found from the evolution equation

$$\dot{a}^2 + 1 = \tfrac{8}{3}\pi G\rho_V a^2 , \tag{3}$$

where $\dot{a} = da/dt$. The solution of this equation is the de Sitter space,

$$a(t) = H^{-1}\cosh(Ht) , \tag{4}$$

where $H = (8\pi G\rho_V/3)^{1/2}$. It describes a universe which is contracting at $t < 0$, reaches its minimum size ($a_{\min} = H^{-1}$) at $t = 0$, and is expanding at $t > 0$. This behaviour is analogous to that of a particle bouncing off a potential barrier at $a = H^{-1}$. (Here a plays the role of the particle coordinate.) We know that in quantum mechanics particles can tunnel through potential barriers. This suggests that the birth of the universe might be a quantum tunneling effect. Then the universe has emerged having a finite size ($a = H^{-1}$) and zero "velocity" ($\dot{a} = 0$); its following evolution is described by eq. (4) with $t > 0$.

Sidney Coleman [9] has taught us that a semiclassical description of quantum tunneling is given by the bounce solution of euclidean field equations (that is, of the field equations with t changed to $-it$). Normally, bounce solutions are used to describe the decay of a quasistable state. If the decaying state is at the bottom of a potential well at $x = x_1$, then the bounce solution starts with $x = x_1$ at $t \to -\infty$, bounces off the classical turning point at the end of the barrier, and returns to $x = x_1$ at $t \to +\infty$.

The euclidean version of eq. (3) is $-\dot{a}^2 + 1 = H^2a^2$, and the solution is

$$a(t) = H^{-1}\cos(Ht) . \tag{5}$$

Eqs. (2) and (5) describe a four-sphere, S^4. This is the well-known de Sitter instanton [10]. The solution (5) does bounce at the classical turning point ($a = H^{-1}$); however, it does not approach any initial state at $t \to \pm\infty$. In fact, S^4 is a compact space, and the solution (5) is defined only for $|t| < \pi/2H$. The instanton (5) can be interpreted as describing the tunneling to de Sitter space (4) from *nothing*. Then the birth of the universe is symbolically represented in fig. 1a.

The concept of the universe being created from nothing is a crazy one. To help the reader make peace with this concept, I would like to give an example of a compact instanton in a more familiar setting. Let us consider the creation of electron–positron pairs in a constant electric field E. For simplicity, we shall work in a (1 + 1)-dimensional space–time. The energy conservation law for the electron is

$$m(1-v^2)^{-1/2} - eEx = \text{const} , \tag{6}$$

where $v = dx/dt$, m and e are electron mass and charge, respectively. The solution of eq. (6) is

$$x - x_0 = \pm[\kappa^2 + (t-t_0)^2]^{1/2} , \tag{7}$$

where $\kappa = |m/eE|$ and x_0, t_0 = const. The classical turning points are at $x = x_0 \pm \kappa$. The instanton solution describing the creation of a pair is obtained from eq. (7) by changing t to $-it$:

$$(x-x_0)^2 + (t-t_0)^2 = \kappa^2 . \tag{8}$$

It describes a circular trajectory, that is, again we have

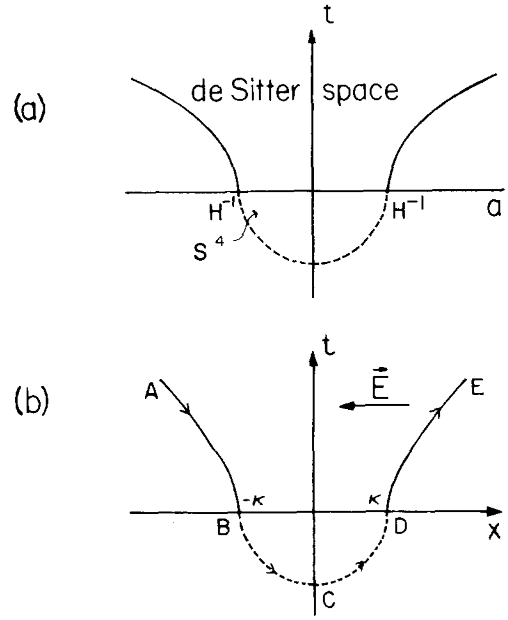

Fig. 1. A schematic representation of (a) birth of the inflationary universe and (b) pair creation in the electric field. In both cases dashed semicircles represent the "under-barrier" part of the trajectory. (Below the horizontal axis t is the euclidean time.) The classical evolution starts at $t = 0$.

a compact instanton. The process of pair production is symbolically represented in fig. 1b. AB and DE are classically allowed trajectories. AB describes an electron moving backwards in time, that is a positron. The semicircle BCD represents the instanton (8). The instanton solution (8) can be used to estimate the semiclassical probability, P, of pair creation per unit length per unit time: $P \propto \exp(-S_E)$, where S_E is the euclidean action,

$$S_E = \int [m(1 + \dot{x}^2)^{1/2} - eEx] \, dt .$$

Introducing a new variable, ϕ, according to $x - x_0 = \kappa \cos\phi$, $t - t_0 = \kappa \sin\phi$, we find

$$S_E = \frac{m^2}{|eE|} \int_0^{2\pi} \sin^2\phi \, d\phi = \frac{\pi m^2}{|eE|} \tag{9}$$

so that $P \propto \exp(-\pi m^2/|eE|)$, a well-known result [11] [‡1].

[‡1] A similar approach to the calculation of the rate of particle production in external field has been used in ref. [12]. I am grateful to A. Guth for pointing this out to me.

Of course, the evaluation of the probability P is possible because the pair creation takes place in a background flat space. The instanton solution contributes to the imaginary part of the vacuum energy. Such a calculation does not make sense for our de Sitter instanton: it is silly to evaluate the imaginary part of the energy of nothing. The only relevant question seems to be whether or not the spontaneous creation of universes is possible. The existence of the instanton (5) suggests that it is. One can assume, as usual, that instantons, being stationary points of the euclidean action, give a dominant contribution to the path integral of the theory. There may be several relevant instanton solutions. For example, we can have a de Sitter instanton with broken grand unified symmetry, but unbroken Weinberg–Salam symmetry. Then the vacuum energy is $\tilde{\rho}_v \sim \sigma_{ws}^4 \ll \rho_v$, where $\sigma_{ws} \sim 100$ GeV is the energy scale of the SU(2) × U(1) symmetry breaking. The euclidean action of a de Sitter instanton is negative [10,13], $S_E = -3m_p^4/8\rho_v$. If one assumes that instanton with the smallest value of S_E correspond, in some sense, to most probable universes, then most of the universes never heat up to temperatures greater than 100 GeV and have practically vanishing baryon numbers. Obviously, we must live in one of the rare universes which tunneled to the symmetric vacuum state.

Finally, we have to discuss what happens to the universe after the tunneling. The symmetric vacuum state is not absolutely stable. It can decay by quantum tunneling [6] or can be destabilized by quantum fluctuations of the Higgs field [7]. The Higgs field starts rolling down the effective potential towards the glorious ending of the inflationary scenario, as it is discussed in refs. [3,4,6] and at the beginning of this paper. When the vacuum energy thermalizes, the universe heats up to a temperature $T_* \sim \rho_v^{1/4}$. In our model this is the maximum temperature the universe has ever had. The only verifiable (in principle) prediction of the model is that the universe must be closed. However, Guth has argued [14] that the inflationary scenario almost certainly overshoots, so that $\rho = \rho_{crit}$ with a very high accuracy even at the present time. This means that we shall have to wait for a long time until the sign of $(\rho - \rho_{crit})$ can be determined experimentally. The advantages of the scenario presented here are of aesthetic nature. It gives a cosmological model which does not have a singularity at the big bang (there still may be a final singularity) and does not require any initial or boundary conditions. The structure and evolution of the universe(s) are totally determined by the laws of physics.

Note added: The possibility of spontaneous creation of closed universes has been first discussed by Tryon [15]. Quantum tunneling of the universe as a whole has been discussed by Atkatz and Pagels [16] and Hawking and Moss [17].

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